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1989 J. Phys. A: Math. Gen. 22 919

(http://iopscience.iop.org/0305-4470/22/7/022)

J. Phys. A: Math. Gen. **22** (1989) 919–920. Printed in the UK

COMMENT

Comment on 'Stokes phenomena and the monodromy deformation problem for the non-linear Schrödinger equation'

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Received 14 January 1988

Abstract. A comment is presented on a recent paper by Chowdhury and Naskar which describes a method that is claimed to be successful in calculating the Stokes parameters of the linear differential equation to which the Painlevé IV equation defines isomonodromic deformations. However, we show that the simplification made by the authors is not uniform and the resulting formulae for Stokes parameters are not invariant under the deformations satisfying the Painlevé IV equation.

1. Introduction

Recently Chowdhury and Naskar [1] claimed that, using the results due to Sibuya [2], they found a method to calculate explicitly the Stokes parameters for the direct problem of the non-linear coupled system of ordinary differential equations:

$$-(\mathrm{d}/\mathrm{d}z)(\phi_z+\mathrm{i}z\phi/2)=\mp 2\phi^2\phi^* \tag{1}$$

which is equivalent to the Painlevé IV equation. The direct problem of the isomonodromic deformation method for Painlevé IV consists of the evaluation of the so-called monodromy data of the corresponding linear system:

$$V_\xi=\begin{vmatrix} F & G \\ G^* & -F \end{vmatrix} V \tag{2}$$

where

$$\begin{aligned} &V^{\mathrm{T}}=(V_1, V_2) && F=4\mathrm{i}\xi+2\mathrm{i}\phi\phi^*\xi^{-1}-\mathrm{i}z \\ &G=-4\phi+(z\phi-2\mathrm{i}\phi_z)\xi^{-1} && G^*=-4\phi^*+(z\phi^*+2\mathrm{i}\phi_z^*)\xi^{-1}. \end{aligned} \tag{3}$$

2. Results

To obtain the Stokes parameters of the linear equation (2), which are a part of the monodromy data, Chowdhury and Naskar attempt to use a method given by Sibuya only for a special kind of second-order scalar ordinary differential equations with polynomial coefficients:

$$Y''=(\xi^2+a_1\xi+a_2)Y. \tag{4}$$

So, to reduce system (2) into a scalar equation of the form (4), one eliminates the second component of the vector V. Defining a new unknown function $W=G^{-1/2}V$, and a new independent variable $\xi' = 2\exp(-i\pi/4)\xi$ (the rescaling defined in equation (33) of [1] is completely insufficient for this purpose), and omitting the primes in ξ', one obtains the second-order scalar ODE of the form:

$$W'' = [\xi^2 + a_1\xi + a_2 + A_1\xi^{-1} + A_2\xi^{-2} + A_3(\xi - B)^{-1} + A_4(\xi - B)^{-2}]W \quad (5)$$

where

$$a_1 = -z\exp(-i\pi/4) \qquad a_2 = -1 - iz^2/4$$

$$A_1 = \exp(i\pi/4)[\phi(1+4i\phi\phi^*)/(z\phi - 2i\phi_z) - 2(z\phi\phi^* - 2i\phi^*\phi_z + 2i\phi\phi^*) - iz/2]$$

$$A_4 = \tfrac{3}{4}$$

and A_2, A_3 and B are also functions of z, ϕ, ϕ^*, ϕ_z and ϕ_z^*. For large $|\xi|$, although it is not uniform in z, it can be justified to truncate the coefficient of the term with W to obtain an approximate equation of the form (4). However, to obtain formulae for the monodromy data of the second-order ordinary differential equation in (4), Sibuya [2] takes the behaviour of the solutions of (4) for small $|\xi|$ as well. But, in this range of the independent variable, it is evident that equation (4) cannot approximate equation (5). Hence, even asymptotically for large $|\xi|$ Sibuya's formulae are not applicable to differential equations of the type (5). Ignoring this fact, and using the formulae of Sibuya, Chowdhury and Naskar calculate Stokes parameters for the ODE in (5). As a result they obtain monodromy data which are not invariant under the deformations satisfying Painlevé IV. As an example they find

$$a = (1/i\phi)c_{-1} \quad (6)$$

for the first Stokes parameter, which is evidently z dependent for functions ϕ which satisfy Painlevé IV.

References

[1] Chowdhury A R and Naskar M 1986 Stokes phenomena and the monodromy deformation problem for the non-linear Schrödinger equation *J. Phys. A: Math. Gen.* **19** 3741–53

[2] Sibuya Y 1974 Uniform simplification in a full neighborhood of a transition point *Mem. Am. Math. Soc.* **149**